LOYOLA COLLEGE (AUTONOMOUS) CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – **MATHEMATICS**

THIRD SEMESTER – **NOVEMBER 2024**

UMT 3502 – DIFFERENTIAL EQUATIONS AND LAPLACE TRANSFORM

Date: 12-11-2024 Dept. No. Max. : 100 Marks

Time: 09:00 am-12:00 pm

	SECTION A - K1 (CO1)	
	Answer ALL the Questions	**(10 x 1 = 10)**
1.	**Answer the following**	
a)	Define Order of a Differential Equation	
b)	Solve $(D^2 - 3D+2)$ y=0	
c)	Eliminate arbitrary constants from $z = ax + by + b$.	
d)	Find $L\{t^2+2t+3\}$	
e)	Find $L^{-1}\left\{\frac{1}{(s+a)^2}\right\}$	
2.	**Fill in the blanks**	
a)	The General Solution of the Differential equation $\frac{dy}{dx}=\frac{1+y^2}{1+x^2}$ is¿.	
b)	The Particular Integral of $(D^2$ +5D+6) y=e^x is _____	
c)	The Complete Integral of pq = k is _____	
d)	$L\{Sinhat\}$=¿¿	
e)	$L^{-1}\left\{\frac{s-3}{(s-3)^2+4}\right\}$=¿¿	
	SECTION A - K2 (CO1)	
	Answer ALL the Questions	**(10 x 1 = 10)**
3.	**Match the following**	
a)	The Integrating factor of the Differential equation $\frac{dy}{dx}+ycosx=\frac{1}{2}\sin 2x$ a) e^{cosx} b) Sinx c) e^{sinx} d) $e^{\sin 2x}$	
b)	The auxiliary equation of $5\frac{d^2y}{dx^2}+7\frac{dy}{dx}+9y=e^{8x}$ is (i)$5m+7m^2+9=0$ (ii) $5m^2+7m+9=0$(iii) $5m^2+7m+9=e^{8x}$(iv) $5m+7m^2+9=e^{8x}$	
c)	The solution of $z = px + qy +2\sqrt{pq}$ is (i) $z = p + q + pq$ (ii) $z = ax + by +2\sqrt{pb}$ (iii) $z = pa + qb + ab$ (iv) $z = ax + by +2\sqrt{ab}$	
d)	$L\left\{\frac{t^3}{3}\right\}$ is (i) $\frac{6}{s^4}$ (ii) $\frac{4}{s^4}$ (iii)$\frac{3}{s^3}$ (iv) (i) $\frac{2}{s^4}$	
e)	$L^{-1}\left\{\frac{1}{(s+3)^2}\right\}$ is a) $e^{3t}t$ b) $e^{-3t}t$ c) $e^{3t}t^2$ d) $e^{-3t}t^2$	

4.	**True or False**
a)	A differential equation is said to be linear when the dependent variable and its derivate occur in first degree only.
b)	D^2 (sinax) is - $-a^2 Sinax$.
c)	The Partial Differential equation of all spheres whose centres lie on the z- axis is xp = yq.
d)	$L\{f'(t)\} is\ s\,L\{f(t)\}$
e)	Laplace transform is used to solve system of differential equations

SECTION B - K3 (CO2)

Answer any TWO of the following in 100 words each. (2 x 10 = 20)

5.	Solve $(1-x^2)\frac{dy}{dx}+2xy=x\sqrt{1-x^2}$ given that y=0 when x = 0
6.	Solve $\frac{d^2y}{dx^2}+4y=\tan 2x$ using variation of parameters.
7.	Solve (y-z) p + (z-x) q = x - y.
8.	Find the Laplace transform of $t\,e^{-t}\,sint$

SECTION C – K4 (CO3)

Answer any TWO of the following in 100 words each. (2 x 10 = 20)

9.	Solve $x\frac{dy}{dx}+y=x^3y^6$.
10.	Solve $x^2\frac{d^2y}{dx^2}-x\frac{dy}{dx}+y=\log x$.
11.	Using Charpit's method, solve $p^2+q^2-2px-2qy+1=0$.
12.	Find the Laplace transform of the Periodic function with period a defined as f(t) = t, $0 \le t < a$

SECTION D – K5 (CO4)

Answer any ONE of the following in 250 words (1 x 20 = 20)

13.	Using Laplace transform, solve the equation $(D^2+2D-3)y=\sin t$ given that $y=0, y'=0 when t=0$.
14.	A body of mass m falling from rest is subject to the force of gravity and an air resistance directly proportional to the square of the velocity (kv^2). If it falls through a distance x and possesses a velocity v at that instant prove that $\frac{2kx}{m}=\log\left(\frac{a^2}{a^2-v^2}\right)$

SECTION E – K6 (CO5)

Answer any ONE of the following in 250 words (1 x 20 = 20)

15.	a)Evaluate $\int_0^\infty t\,e^{-2t}\cos t\,dt$ (10 Marks) b) Verify that $\int_0^\infty \frac{e^{-t}-e^{-2t}}{t}dt$ = log2. (10 Marks)
16.	Solve the simultaneous equation: $3\frac{dx}{dt}+\frac{dy}{dt}+2x=1$, $\frac{dx}{dt}+4\frac{dy}{dt}+3y=0$, given that x = y = 0 at t = 0.

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